Problem: American Elk

Prior to the arrival of European colonization on the North American continent, the ecological bio-diversity was much richer than we currently know in the 21st Century. Prior to the colonization animals such as the American Bison (Bison bison), Eastern Elk (*Cervus canadensis canadiensis)*, Eastern Cougar (*Puma concolor couguar*), and Wolf (*Canis lupus)* were commonly seen across the North American continent. However, with the colonization from the old world came old world prejudices and practices. Within two hundred and fifty years all of these species were eradicated from the Eastern United States or extinct.[1] Over the course of the last century an effort has been made to halt the loss of the North American fauna with the creation of national parks and animal preservation habitats, and in the last half of 20th century work was being done to reintroduce that fauna back to its natural habitat. This process has been used with several different species; however, no species has been given as much attention in the Eastern United States as that of the American Elk.

Prior to the reintroduction of Elk back into Eastern United States the only attempt at having these animals in their natural Eastern habitat was done by the owners of private exotic hunting preserves. However, in the latter half of the 20th century serious studies were conducted on the impact of a reintroduction program. The problems listed for this reintroduction program were as follows:

- What states to reintroduce the elk
- What would be the impact on agriculture
- Would the Elk adapt to the more densely populated Eastern U.S.

However, the most pressing question was the elk species themselves. The Elk native to the Eastern U.S. (*Cervus canadensis canadiensis)* was hunted to extinction sometime in the early 1800s. The Eastern Elk, while similar to the Manitoba Elk (Cervus *canadensis manitobensis)*, are not the same. The Manitoba Elk were smaller in size than the Eastern subspecies and were adapted to living in the Western U.S and Canadian prairie. These adaptations included disease tolerances, foods, and environmental differences. How would these adaptations affect an introduction of Manitoba Elk into the East and more specifically in the Great Smokey Mountain National Park (GSMNP)?

Build a mathematical model to determine whether the elk survive or die out. Regardless, come out up with a plan to improve the growth of the elk over time. In addition to your one page summary sheet and complete project report prepare a one page letter describing your results to the Commissioner of the Department of Wildlife. The following are the reported numbers over the course of the study:

Year	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
Reintroduced	25	27					5				
Births	4	2	10	8	10	13	19	19	19	25	19
Death											
Poached				1					1	1	
Sickness	1	5	2	6	2			3	2	3	
Accident			1			1	1	1		2	
Predator	1		1	2	5	1	4		2		
Unknown				3				5	3	2	3
Population	27	51	57	53	56	67	86	96	107	124	**140**

Figure 1: This is the numeric data for the population growth and the corollary effects on the growth, culminating into the current approximate population of the herd in the GSMNP region.

[1] This is an approximation. Some species were killed off faster than others, with the Eastern Elk being killed to extinction in the early 1800s and the Eastern Cougar being recorded as being killed off as late as 1930s. Ref: http://www.huffingtonpost.com/2011/03/02/eastern-cougar-extinct-mo_n_830181.html and http://www.rmef.org/Conservation/HowWeConserve/Restoration/FallRise/

Summary

Prior to European colonization of America, biodiversity, more specifically the native elk population, was far greater than the degree it is now. Ecologists planned to introduce Manitoba Elk, indigenous to the Western U.S. and Canadian prairie, into the Great Smoky Mountain National Park of the East coast in order to revitalize eradicated elk populations and stimulate biodiversity. We were charged to create a model predicting population dynamics of the previously introduced Manitoba elk and to develop a feasible plan for maintaining a stable population with minimal risk of endangerment.

In order to account for factors which determine survival rate and population changes, our team used a Density-dependent or Logistic model. To determine the carrying capacity of an environment we created a comprehensive equation based on the minimum acreage required for an individual elk's survival. Ruling out the less significant factors such as innate characteristics of Manitoba elk, discrepancies in abiotic environmental factors, variations in types of food, possible TB and CWD transmission, and changes due to migration for greater emphasis rather on factors including changes in population over a given period of time, presence of other competitors for limited resources, and presence of predators in contribution to mortality rate to determine our Logistic models.

We then tested our model with legitimate research data on the previously recorded elk population dynamics in GSMNP over a 10 year period (2001-2011). Using this data and our own analysis we were able to establish birth and death rates for the introduced elk population and thus project the future survivability and sustainability of this population. Incorporating further constants including carrying capacity, initial population, and reproductive potential we were able to create a comprehensive model for the population of the elk within the GSMNP.

Because the current population of the elk in the GSMNP is capable of surviving without further human intervention, we decided that no additional populations of elk need to be introduced over time to maintain its maximum population. Our plan to increase the population of the elk would be to allow the elk to continue to propagate naturally without risks of endangerment. Thus, we proposed implementing a number of reinforcement measures within the National Park, including prohibiting poaching indefinitely and monitoring when the population is at risk so that park ecologists could plan to alter the resources available or reduce interspecies competition based on our additional plans.

November 11, 2012

US Department of Wildlife
1801 N. Lincoln Blvd.
Oklahoma City, OK, 73105

Dear Commissioner of the Department of Wildlife,

Using accurately modeled and carefully applied data of integration techniques for the Manitoba elk, *cervus candensis manitobensis* species, Team 3797 has concluded that within a timely fashion, they will soon roam the Eastern United States numbers populous yet controlled – or at least within the boundaries of the Great Smoky Mountains National Park. Though the previous Eastern Elk, *Cervus canadensis canadensis* were driven to extinction in the 19th century, our analysis of the logistic model indicates that the western Manitoba species are expected to thrive just as adequately if not more so. This newly introduced American elk will benefit the ecosystm, contributing to the East coast's biodiversity. ,

And the reinforcement measures for this experiment? The key to maintaining a thriving Manitoba elk population within the GSMNP lies in prevention of what had formerly driven the previous inhabitants, the Eastern Elk, to extinction – over-poaching. Since population growth is limited by deaths, we can increase quantity by either eliminating elk predators from the region by translocating them or prohibiting poaching entirely. Though the GSMNP prohibits poaching of any form without registered license, any animals found outside of the park's designated boundaries can be legally shot. Because the GSMNP does not have the authority to expand its boundaries to encompass wherever the elk may roam, we propose the usage of nonlethal physical barriers to limit their foraging area to within the park for their own protection and social media to increase public awareness of the elk whilst in critical stages of growth. Our preventative approach is both efficacious and cost effective, as it utilizes regulation, using an "ounce of prevention" rather than a far more costly cure.

Though we expect the elk population to thrive, fluctuations will occur due to unaccounted for factors including weather, and natural disaster. These however, should not play a vital role in their success.

We expect the elk population to rebound quickly without fail.

Sincerely,
COMAP Group 3797

2012 HiMCM Problem A
American Elk
Team 3797

Table of Contents

Problem A: American Elk

Introduction

North America was once home to one of the most diverse ecosystems in the world. Many animals such as the elk which once roamed the eastern United States and the Appalachian mountains disappeared in a few hundred years following the European colonization of America. The European colonization contributed to excessive hunting and loss of habitat of the elk. In the early 1900s, after the elk had all but disappeared from several eastern states did hunting groups and conservation groups begin to advocate for their protection.

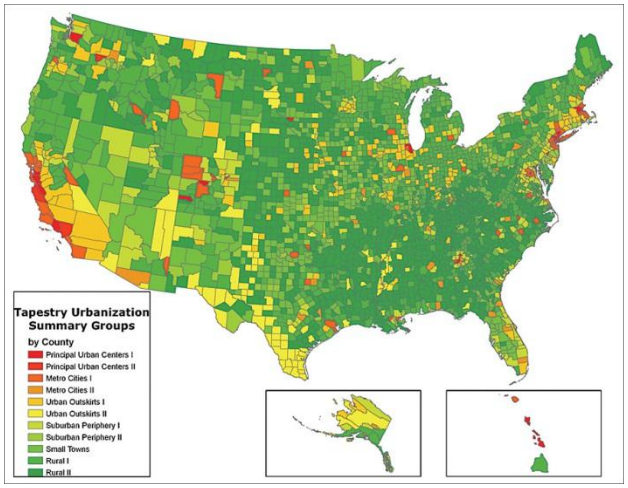
Today, the elk populations still remain low within the eastern United States. Efforts to introduce the Manitoban elk (*Cervus canadensis manitobensis)* in the areas previously dominated by the now extinct Eastern elk are currently investigated by the National Park Service. Despite the honorable intentions of restoring biodiversity to the east coast, simply introducing Manitoban elk could very well end in failure. The Manitoban species are not only smaller than the Eastern species, but they are also adapted to different diseases, foods, and ecosystems. In this paper we address the plausibility of sustaining the Manitoban elk in the Great Smoky Mountain National Park through a mathematical model of population growth and sustainability.

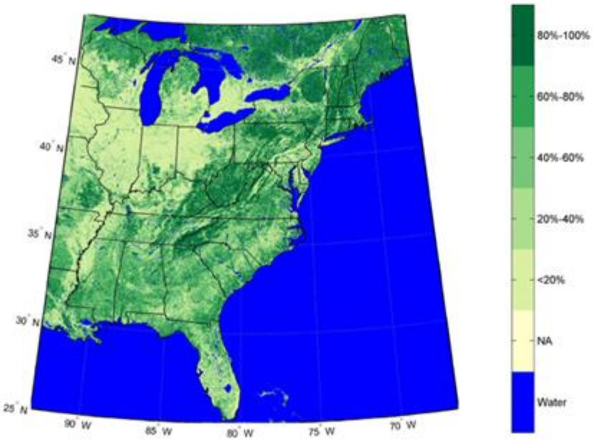
Interpretation / Problem Restatement

By means of a mathematical model, address the following issues encountered by previous reintroduction programs; what states to reintroduce elk, the elk's impact on agriculture, and whether the elk would successfully adapt to the more densely populated Eastern U.S. or not. (Pt. 1 abc) In addition, determine the survival success or failure of the Manitoban Elk species introduced to the Great Smoky Mountains National Park. (Pt. 2) Address factors inherent to the species that may affect their survival and also incorporate the previous data provided from the study. (Pt. 3)

Assumptions/Justifications

Assumption 1 - The Eastern United States is defined as the Appalachian Mountains and eastward (not including Florida).
Justification - Although the Eastern United States is often defined as the east of the Mississippi River, we found that the areas east of the Appalachian mountains had the most homogenous geography and that a larger area would contain too many geographical factors that are impossible to approach.
Assumption 2 - The geographical regions we introduce these elk have enough resources to provide sustainable conditions to at least identify the carrying capacity and survivability success or failure of the Manitoba elk.
Justification - The Manitoban elk may be adapted to differing environments however in order to identify where their possible integration will succeed or fail we must be able to determine the K-value for our logistic model obtained either through calculations by ourselves or other researchers in the legitimate field of ecology.





Urbanization in the US (Green denotes rural areas)   Tree Cover in the Eastern US

Assumption 3 - There will be no significant emigration or immigration of the elk.
Justification - Emigration, Immigration, and migratory patterns would be near impossible to accurately model without actual field tracking over a period of time.
Assumption 4 - Differences between individuals such as sex, age, breeding status, state of health, etc. are ignored.
Justification - Individual of the Manitoban species are so varied that we found it impossible to incorporate every factor effectively in the model.
Assumption 5 - Population cycles will be disregarded.
Justification - Fluctuations in population throughout the year cannot affect the model because it is not time based but based on the population given at the time.

The Mathematical Models Pt. 1

A. What states to introduce the elk?

In order to maximize efficiency for identifying feasible locations for further elk relocation and integration, we decided creating logistic models should be based on geographical and environmental characteristics per each ecoregion rather than state lines drawn by the US government regardless of nature. Thus we decided to use the Density-dependent or Logistic model for the following 3 ecoregions; the North American Desert, Great Plains, and Eastern Temperate Forests. In calculating the variables for the Logistic model however, we must calculate the individual K-values for carrying capacity as each ecoregion can sustain differing amounts of elk depending upon the available food resources and present number of competitors. All these variations per each ecoregion however, make it near impossible to determine the value for K given our inability to conduct field research on our own. Even if we did attempt to calculate carrying capacity per state, this too would be implausible without sufficient data from previously conducted research studies. Due to this lack of necessary information, we have deemed where to place the elk for population success as inconclusive.

B. What would be the impact on agriculture?

When elk herds expand their foraging range to include agricultural products cultivated for human consumption, we find that there are significant losses in alfalfa, wheat, and sunflower fields proportional to the number of elk. As a result however, farmer resentment tends to lead to poaching and the herd's numbers fall back.

C. Would the Elk adapt to the more densely populated Eastern U.S.?

The Manitoba elk's new environment in the eastern U.S. has a higher population density than its indigenous environment in the west, meaning that the amount of resources available to each individual is far lower. Consequently, the carrying capacity for the Manitoba elk in the east is slightly lower in comparison to its carrying capacity in the west. This slight difference is not significant because, by analyzing scientifically researched data through our model, we found that the carrying capacity in eastern ecosystems is sufficient for Manitoba elk to survive after their translocation to the east.

The Mathematical Models Pt. 2

Density-Dependent (Logistic) Model

$$P(t) = \frac{K P_0 e^{rt}}{K + P_0 \left(e^{rt} - 1\right)} \text{ where } \lim_{t\to\infty} P(t) = K.$$

P(t) = change in population over period of time
P0 = number of individual elk in initial population
r = biotic potential or reproductive capacity of individual (probability of reproduction)
t = number of years passed since introduction of initial population
K = carrying capacity

Carrying Capacity Equation

A = total available acreage
C = acres taken by native competitors = minimum acreage per individual x population
a = minimum acreage required per elk herd

We derived a comprehensive formula for calculating carrying capacity based on amount of resources available within a certain amount of land. In general there are two ways of calculating carrying capacity; 1. How much biomass the environment has to offer and 2. Minimum acreage necessary for survival for each individual elk. Since calculating the biomass of the environment would have to take into account the varying biomass of each possible food sources based on their geographic density, we found it far easier to instead create our equation based on the minimum acreage necessary for survival. After moderate research, we were able to discover the minimum number of acres necessary for elk specifically in the GSMNP – 4500 acres per elk. For further elaboration on application of this equation to the national park see K value below under specific variables/parameters for GSMNP.

Analysis using Actual Data

Specific variables/parameters for GSMNP
Po = number of individual elk initially introduced into the park since 2001 = 140
r = 0.0924 (biotic potential or reproductive capacity of individual)

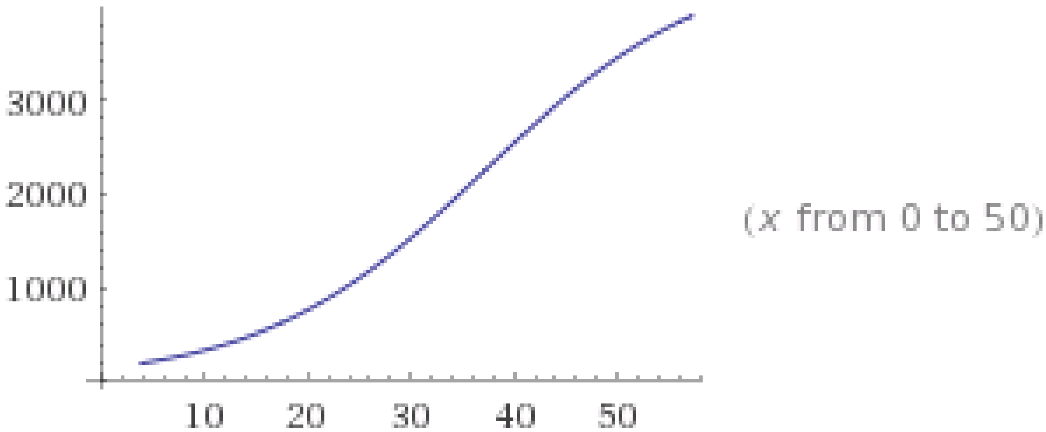
Assuming that there is a 23:77 sex ratio of males to females and average 12% of the females in pregnancy at any given time during the year. We can calculate an overall probability of .0924 that any suitable female elk (for the purposes of simplifying our model) would be able to successfully bear offspring. (0.77x 0.12= 0.0924)

K = 4500 (carrying capacity in # of individual elk)

The carrying capacity of the Great Smoky Mountain National Park is around 104 herds. This was determined using the minimum habitat area of 4,500 acres per herd as the park is 522,419 acres. If competition coming from other mammals currently inhabiting the area (white-tailed deer) take up to some 250 acres per adult individual while there are some 500 deer within the park. (Though deer require a substantially larger foraging area, the overlapping sources of food for elk only consist 250 acres of their minimal 700 acreage) [522419 - (250x500)] / 4500 = 88.315 herds x 50 individual elks per herd thus the K value or maximum carrying capacity of GSMNP is around 4500 Manitoba elk.

t = number of years

The variable t is the independent variable (x-axis) in this equation, since we are seeing the effects of the environment's current conditions on the current population over time.



P(t) = (630000e^(0.0924t))/(4500+140(-1+e^(0.0924t))

where x-axis represents t in increments of years and the y-axis represents the total number of Manitoba elks in the population

Proof of survival success

Since the carrying capacity of the population is significantly higher than the initial population, the slope of the population function is positive, never negative. In application, the elk population's net change is constantly increasing, not decreasing. By using this density-dependent or logistic model, we can safely conclude that the current population of 140 Manitoba elk in the Great Smoky Mountain National Park will survive over time and eventually reach the carrying capacity of the environment. We can further prove that though the death rate may decrease the population, the birth rate (b) is greater than the death rate (d) so the difference between the two can be determined as (bN – dN = # births - # deaths = positive value). Where N = number of elks in the population at the time.

Number of Births:

Typically adult elk populations will have a 23:77 sex ratio of male to female elks. Of the 77% females, an average 12% of them will be pregnant at any given time during the year. Thus of the total elk population, (.77 x .12 = .0924) 9.24% will bear offspring. The success however, is determined by multiplying the birth rate by the population of elk by the probability of survival for the elk calves. This gives us our equation for births, Y = (.0924 x N x .85) = .07854N

Number of Deaths:

Since long-term comprehensive studies typically do not account for cause-specific mortalities of elks (number of deaths caused by accident, poachers, sickness, or unknown), we found that since the percentage of mortalities caused by predation was on average 66% of all deaths, we used this ratio to calculate the total number of deaths of the population. Since some 8% of a population consist of calves at any time and their typical death rate is 15%, we can conclude that 0.012N represents the rate of calf death in an elk population (0.08N x 0.15 / 0.66 = 0.01818 N). For adult elks which make up the other 92% of the population, regardless of sex and time of year, had an average mortality rate of 4.5%. We can thus calculate that 0.0598N represents the mortality rate of adult elks. (0.92N x 0.03 / 0.66 = 0.041818 N). For the total mortality rate however, we combine the mortality rate of calves and adults to get 0.43636N as our final result. (0.01818N + 0.041818N = 0.043636N)

birth rate - death rate = 0.07854N - 0.043636N = 0.034904N thus there is a gradual increase of 3.4904% of the population in elks per year.

Because the difference in population per year is a positive rather than negative quantity, we can safely conclude the Manitoba elk population in the GSMNP will continue to rise and within some 50 years by around 2060, the number of elks will have near reached carrying capacity. (by 2060, there is an estimated 4180 elks to be found within the park, 320 elks short of carrying capacity)

Reinforcement Measures:

The key to maintaining a thriving Manitoba elk population within the GSMNP or general vicinity lies in prevention of what had formerly driven the previous inhabitants, the Eastern Elk, to extinction – excessive, over-poaching drove them to complete eradication. Since population growth is limited by the number of deaths, we can decrease the quantity by either eliminating elk predators from the region by relocating them elsewhere or prohibit poaching entirely. Though the GSMNP prohibits poaching of any form without registered license, any animals, including elk, found outside of the park's designated boundaries can be legally shot under the law. Because the GSMNP does not have the authority to expand its boundaries to encompass wherever the elk may roam, we propose the usage of non-lethal physical barriers to limit their foraging area to within the park for their own protection and social media to increase public awareness of the Manitoba elk whilst in their critical stages of growth for the first several years. In addition to the proposed measures, we acknowledge the following possibilities for their survival enhancement; increasing their foraging supply by cultivating plants as possible food sources or decreasing the competing cervid species such as white-tailed deer for further food availability.

Though proposed, these courses of action should be implemented only once either a drastic decrease in Manitoban population is confirmed (not a mere fluctuation) or prolonged slowing of population growth while their species total is still far from carrying capacity (or other designated quantity) is recorded. To do this however, a portion of elk must be traceable and identifiable at any time through collars with radio transmitters and population surveys typically via helicopter counting.

Though we fully expect the elk population to thrive and reach optimal quantity wherever they may be, fluctuations will occur due to unaccounted for factors including weather, natural disaster, etc. These however, should not play a vital role in their success and thus expect the elk population to rebound quickly without fail.

The Mathematical Models Pt. 3

Endogenous variables: dependent factors our study is designed to focus on

1. *Change in population over a given period of time*

Reasoning - In order to determine whether the elk population successfully survives or not can only be identifiable over a prolonged period of time spanning perhaps multiple elk populations. Only after multiple cycles of birth and death will we be able to see a developing trend in the population, whether positive or negative is dependent upon the animals and environmental situation themselves.

2. *Presence of other competitors for resources such as food availability.*

Reasoning - The amount of food competitors consume is an integral part of calculating the carrying capacity for our model since as the number of competitors increases, the number of possible elk the environment is capable of sustaining goes down.

3. *Presence of predators in contribution to mortality.*

Reasoning - Since predation is the cause for elk mortality 2 out of 3 times, we considered this as another factor vital to our model in that as the number of predators increases, so does the mortality rate of the elk population.

Exogenous variables: independent factors our study acknowledges but does not take into account within our data and models

1. *Adaptations and innate characteristics of the Manitoba elk*

Reasoning - The effects of these on the population's survival rate are not significant enough. The climate of the eastern United States is not significantly different enough to affect the survival rate of the reintroduced elk. Individuals are almost certainly able to adapt to the new climate more moderate climate. In addition, the foods available for the elk in the eastern United States are similar if not the same as those in their native regions. Lastly, predators of the elk in their native region and in the eastern United States are similar and thus the elk is already adapted to deal with them.

2. *Discrepancies in abiotic factors of an environment including weather, soil, nutrients, etc.*

Reasoning - if we were able to obtain the comprehensive data for each factor and given adequate time, we would be able to account for each one and its impact on the newly introduced Manitoba

3. *Variations in type of food fit for consumption*

Reasoning - The vegetation of the native regions of the western Manitoba elk and the eastern United States are of similar consistencies. In addition, the eastern United States offers an immense variety of plant material edible to these elk. We believe reintroduced individuals would find themselves quick to adapt to a "new" diet that consists of food similar to their "old" diet.

4. *Possible disease transmission from domestic animals such as Bovine Tuberculosis (TB) or Chronic Wasting Disease (CWD)*

Reasoning - TB is located in geographical regions predominant on the west coast - it is near unheard of on the eastern coast. Chronic Wasting Disease is also negligible since previously recorded cases were not located within vicinity of the GSMNP or East coast. In addition, all imported elk whether in 2001 or in the future are assumed to have undergone rigorous veterinary inspection so as to prevent the introduction of either TB or CWD into the Manitoba elk population on the East.

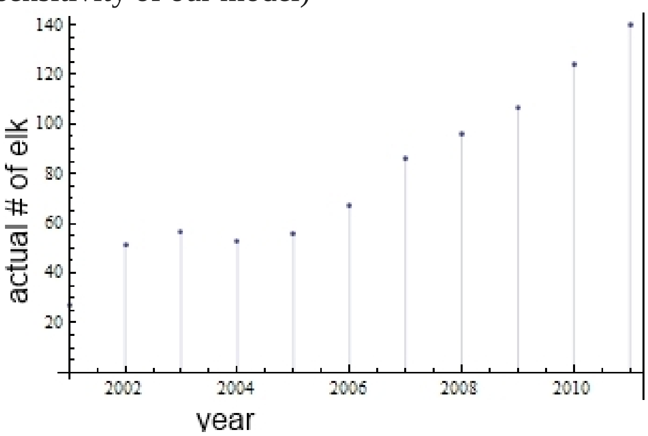
5. *Changes in population due to immigration and emigration*

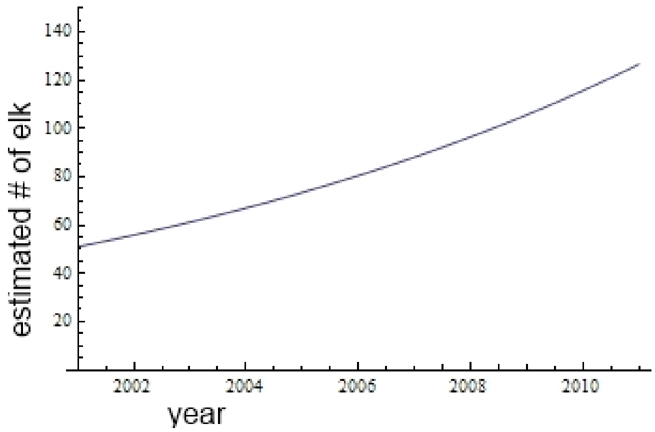
Reasoning - Unless we have the proper tools to track the movement of these elk herds over a prolonged period of time or the data to calculate these results, we assumed that the elk in GSMNP are a closed population while the elk populations elsewhere

Sensitivity Testing

Our model is, at its best, a model, so it accuracy can be affected by several outside factors that were not taken into originally taken into account. However, we are able to easily adjust for certain factor changes. If there is a large number of predators, the death rate of the elk would be adjusted accordingly by increasing. Similarly, if within an ecosystem death often occurs within the elk population from disease, we could easily adjust the death rate and account for the situation. However, our model is only able to address the generalities of an ecosystem and is thus sensitive to individual events that affect the population significantly - such events could be sudden epidemic or a sudden increase in hunting.

In order to determine the accuracy of our model, we were able to compare our model data with the data established through the first 10 years of elk introduction. Although we were not able to account for the addition of reintroduced elks in our model, one can clearly see the consistency our model otherwise has with the GSMNP data. (In the following simulations we have changed the carrying capacity of the ecosystem to 5,200 in order to exemplify the sensitivity of our model)



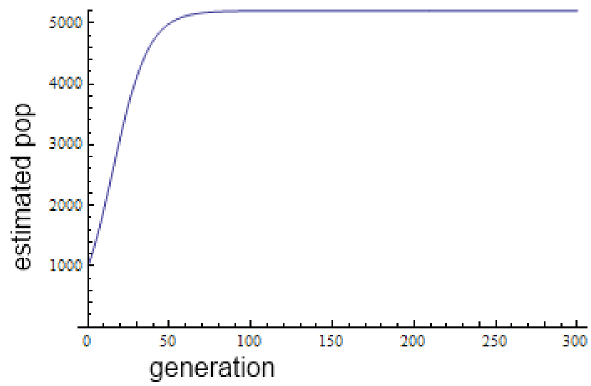


(Model manipulated to start at 2001 in order to compare with data; the introduction of elk in 2001 and 2007 are not consider for the purposes of comparison)

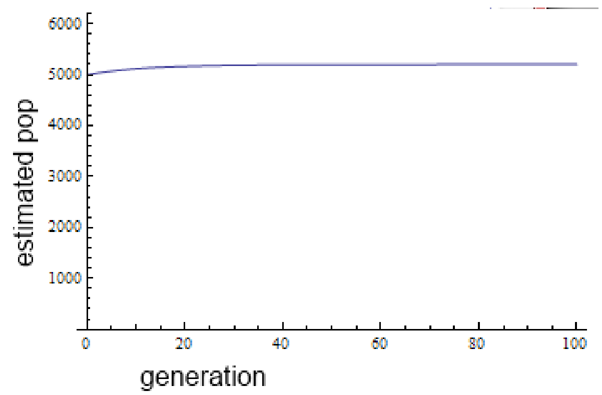
If we start the model in 2002, when the bulk of the elks were reintroduced, our model projects that there be approximately 110 elk in 2011 – an underestimate of the true data. In fact, after about 2008, our model actually becomes an underestimate in terms of the actual data, implying that the propagation and survival of the elk may actually be better than what our model predicts.

However this is only a small interval of actual recorded data – if we were to obtain the statistics of the elk from 2001 to 2060, we would have a much better idea how much error there is within the model. Despite this drawback, we are still able to see consistency between the data and our model – showing that our model may be viable for accurately predicting the success of the deer population in question.

In our model we are able to adjust several factors that can affect the survivability of the population. In the following simulations we have changed the carrying capacity of the ecosystem to to 5,200 in order to show sensitivity of our model.



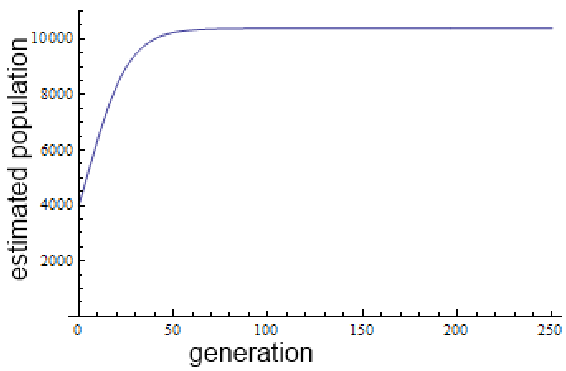
(Initial population 1000)



(Initial population 4000)

The simulations above account for differences in initial population. If we use the birth and death rates established for the GSMNP, we can see that these populations are both stable and survivable.

If we look beyond the GSMNP dynamics and examine another ecosystem with quadruple the area but half the resources per acre of the GSMNP, we can see that our model accommodates such a situation quite nicely. We would simply adjust the carrying capacity to 20,800 from 5,200 and then decrease the 20,800 to 10,400 to account for fewer resources. In addition, if this new ecosystem has say, perhaps, 4,000 individuals - we can easily input this into our model and see the growth of that distinct population.



We also can further address the differences in the ecosystem by adjusting variables such as the birth rate or death rate. If this hypothetical ecosystem had significantly more predators - our model would account for that by adjusting the death rate of the elk.

Strengths of Our Model	***Weaknesses of Our Model***
1. The logistic model is capable of calculating population growth in a wide variety of possible ecosystems to a high degree of accuracy. **2.** Our model is highly adaptable in terms of the birth rate, death rate, and starting population. If there was an epidemic or a high probability of predation among elks, we would	**1.** We do not take into account the possibility of emigration or migration and its effects. **2.** The variability between individuals is not accounted for. **3.** The model is sensitive to sudden introduction of factors that affect population. **4.** The introduction of new elks following an

be able to simply adjust our numbers in order to model the new situation accurately. In addition, we can not only adjust the carrying capacity but also adjust the starting population in order to better account for different ecosystems.	initial group cannot be accommodated for by our model. **5.** Our calculations for carrying capacity of ecosystems are estimated.

Conclusion

Based upon our model we discovered that the current population of the elk in the GSMNP is sustainable and survivable without the introduction of any more elk to the park. The population will increase exponentially and taper off when it reaches the carrying capacity of the park's ecosystem - thus our plan to increase the population of the elk would be to allow the elk to continue to propagate naturally. To ensure that the elk do not experience additional threats, we propose implementing a number of reinforcement measures within the National Park, including prohibiting poaching indefinitely and monitoring when the population is at risk so that park ecologists could plan to alter the resources available or reduce interspecies competition.

In addition, we found that the concerns regarding the adaptations of the Manitoban elk to be insignificant to the end goal of achieving a stable population in the GSMNP. The changes in diet, climate, and geography were not large by any means, so we found reason to believe that the Manitoban species would not have any problems populating the areas in the eastern United States. The majority of region is rural land and any urban areas would just simply be avoided by the elk.

In the future, we wish to model the survivability of elk populations outside of GSMNP. In With that being said, we feel that we could use our model to help implement and model a more widespread and comprehensive elk reintroduction plan on the east coast.

Also, further investigation on our model could include using more nuanced methodology in developing the carrying capacity, births, and deaths of a population. Doing so would provide a much more accurate and realistic model than we have currently proposed.

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